MATHS

$(a+b)^2$

ab+

Some Application of Trigonometry

1. **Trigonometric Ratios**

   Ratio of the sides of a right triangle with respect to the acute angles is called the **trigonometric ratios** of the angle.

   Trigonometric ratios of acute angle A in right triangle ABC are given below:

   $$\text{Sin}\angle A = \frac{\text{side opposite to}\angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{p}{h}$$

   i. $$\cos\angle A = \frac{\text{side adjacent to}\angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{b}{h}$$

   ii. $$\tan\angle A = \frac{\text{side opposite to}\angle A}{\text{side adjacent to}\angle A} = \frac{BC}{AB} = \frac{p}{b}$$

   iii. iv. $$\text{cosec }\angle A = \frac{\text{hypotenuse}}{\text{sideopposite to}\angle A} = \frac{AC}{BC} = \frac{h}{p}$$

   iv. $$\sec\angle A = \frac{\text{hypotenuse}}{\text{side adjacent to}\angle A} = \frac{AC}{AB} = \frac{h}{b}$$

   v. vi. $$\cot\angle A = \frac{\text{sideadjacent to}\angle A}{\text{side opposite to}\angle A} = \frac{AB}{BC} = \frac{b}{p}$$

   The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

2. **Relation between trigonometric ratios**

   The ratios cosec A, sec A and cot A are the reciprocals of the ratios sin A, cos A and tan A respectively as given:

   i. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

   ii. $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

   iii. $\sec\theta = \dfrac{1}{\cos\theta}$

   iv. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

3. **Values of Trigonometric ratios** of some specific angles:

$\angle$A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

4. **Trigonometric ratios of complementary angles**
Two angles are said to complementary angles if their sum is equal to 90^o. Based on this relation, the trigonometric ratios of complementary angles are given as follows:
   i. $\sin(90^o - A) = \cos A$
   ii. $\cos(90^o - A) = \sin A$
   iii. $\tan(90^o - A) = \cot A$
   iv. $\cot(90^o - A) = \tan A$
   v. $\sec(90^o - A) = \operatorname{cosec} A$
   vi. $\operatorname{cosec}(90^o - A) = \sec A$

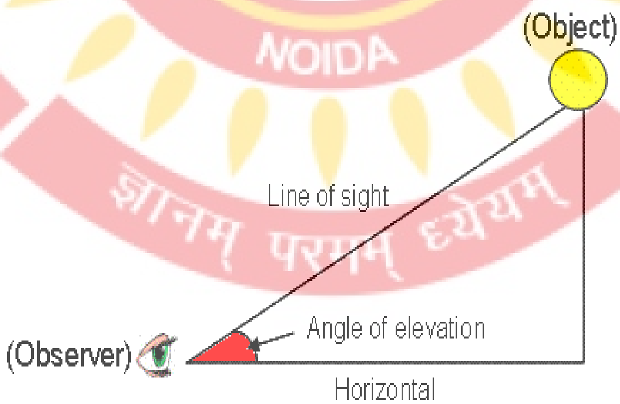
Note: $\tan 0^o = 0 = \cot 90^o$, $\sec 0^o = 1 = \operatorname{cosec} 90^o$, $\sec 90^o$, $\operatorname{cosec} 0^o$, $\tan 90^o$ and $\cot 0^o$ are not defined.

5. **Basic trigonometric identities**:
   i. $\sin^2\theta + \cos^2\theta = 1$
   ii. $1 + \tan^2\theta = \sec^2\theta$
   iii. $1 + \cot^2\theta = \sec^2\theta$

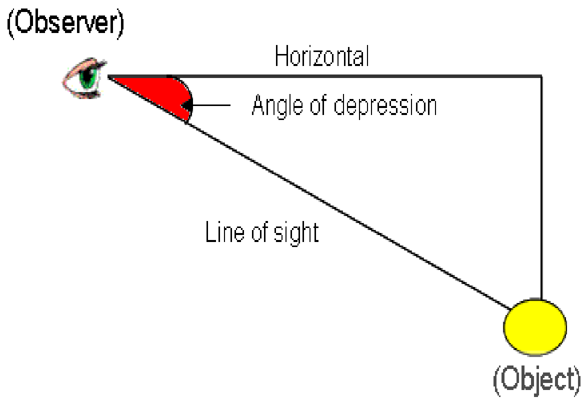
6. The height or length of an object or the distance between two distant objects can be determined by the help of **trigonometric ratios**.

7. **Line of sight**
The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

8. **Angles of elevation and depression**
   - The **angle of elevation** of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.



   - The **angle of depression** of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.



9. Pythagoras theorem

It states that "In a right triangle, square of the hypotenuse is equal to the sum of the square of the other two sides".

- When any two sides of a right triangle are given, its third side can be obtained by using Pythagoras theorem.

10. Reflection from the water surface

In case of reflection from the water surface, the two heights above and below the ground level are equal in length.

MATHS

Pair of Linear Equations in Two Variables

1. **A pair of Linear Equations in two variables:**

   - An equation of the form ax + by + c = 0, where a, b and c are real numbers, such that a and b are not both zero, is called a **linear equation in two variables**.
   - Two linear equations in same two variables x and y are called **pair of linear equations in two variables**.

2. The **general form** of a pair of linear equations in two variables is

   $a_1x + b_1y + c_1 = 0$
   $a_2x + b_2y + c_2 = 0$

   where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$

3. A system of linear equations in two variables represents two lines in a plane. For two given lines in a plane there could be three possible cases:
   i. The two lines are intersecting, i. e., they **intersect at one point**.
   ii. The two lines are **parallel**, i.e., they do not intersect at any real point
   iii. The two lines are **coincident** lines, i.e., one line overlaps the other line.

4. A system of simultaneous linear equations is said to be
   - **Consistent**, if it has **at least one solution.**
   - **In-consistent**, if it has **no solution.**

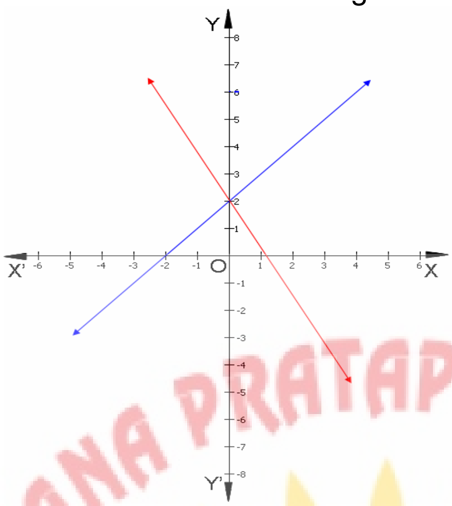
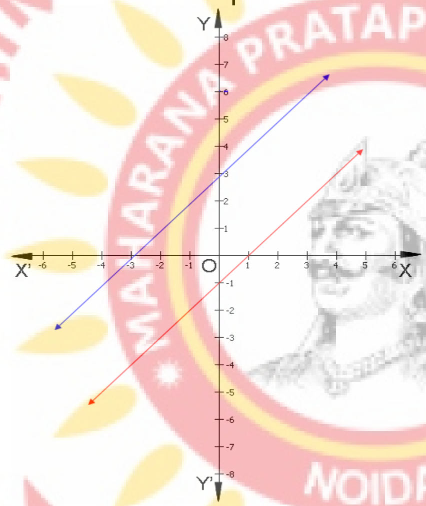
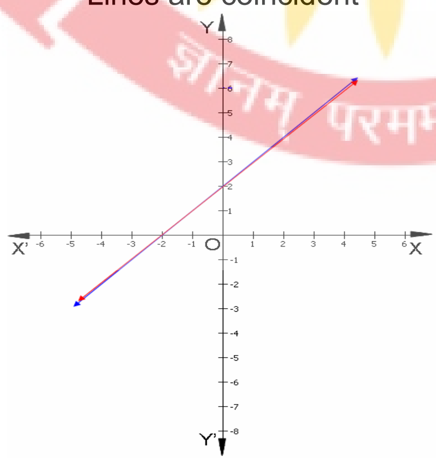
5. If the lines
   i. Intersect at a point, then that point gives the **unique solution** of the system of equations. In this case system of equations is said to be **consistent**.
   ii. Coincide (overlap), then the pair of equations will have **infinitely many solutions**. System of equations is said to be **consistent.**
   iii. are parallel, then the pair of equations has **no solution**. In this case pair of equations is said to be **inconsistent.**

6. **Solution of a pair of Linear Equations in two variable:**

   System of equations can be solved using **Algebraic** and **Graphical Methods.**

7. **Graphical Method:**

   - A linear equation in two variables is represented geometrically by a **straight line**.
   - The graph of a pair of linear equations in two variables is represented by two lines.
     Steps:
     i. Draw the graphs of both the equations by finding two solutions for each.
     ii. Plot the points and draw the lines passing through them to represent the equations.
     iii. The behaviour of lines representing a pair of linear equations in two variables and the existence of solutions can be summarised as follows:

Ratio of Coefficients	Graphical Representation	Nature of Solution	Defined as
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Lines are intersecting 	Unique solution	Consistent pair of equations
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Lines are parallel 	No solution	Inconsistent pair of equations
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Lines are coincident 	Infinitely many solutions	Dependent (consistent) pair of equations

8. Algebraic Method:

The most commonly used **Algebraic Methods** to solve a pair of linear equations in two variables are:

i. Substitution method
ii. Elimination method
iii. Cross-multiplication method

9. Substitution Method:

Steps followed for solving linear equations in two variables, using **substitution method**:

Step 1: Express the value of one variable, say y in terms of other variable x from either equation, whichever is convenient.

Step 2: Substitute the value of y in other equation and reduce it to an equation in one variable, i.e. in terms of x. There will be three possibilities:

a. If reduced equation is linear in x, then solve it for x to get **a unique solution.**
b. If reduced equation is a true statement without x, then system has **infinite solutions.**
c. If reduced equation is a false statement without x, then system has **no solution.**

Step 3: Substitute the value of x obtained in step 2, in the equation used in step 1, to obtain the value of y.

Step 4: The values of x and y so obtained is the coordinates of the solution of system of equations.

10. Elimination Method:

Steps followed for solving linear equations in two variables, by **elimination Method**:

Step 1: Multiply both the equations by some suitable non-zero constants to make the coefficients of variable x (or y) equal.

Step 2: Add or subtract both the equations to eliminate the variable whose coefficients are equal.

a. If an equation in one variable y (or x) is obtained, solve it for variable y (or x).
b. If a true statement involving no variable is obtained then the system has **infinite solutions**.
c. If a false statement involving no variable is obtained then the system has **no solution**.

Step 3: Substitute the value of variable y (or x) in either of the equation to get the value of other variable**.**

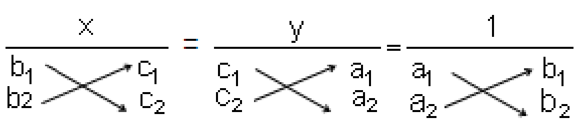
11. Cross Multiplication Method:

Steps followed for solving linear equations in two variables, by **cross multiplication method**:

Step 1: Write the equations in the general form.

$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$

Step 2: Arrange these in the following manner.



Here, the arrows between two numbers (coefficients) mean that they are to be multiplied and the second product is to be subtracted from the first product.

Step 3: Cross multiply:

$$\underset{(1)}{\frac{x}{b_1c_2 - b_2c_1}} = \underset{(2)}{\frac{y}{a_2c_1 - a_1c_2}} = \underset{(3)}{\frac{1}{a_1b_2 - a_2b_1}}$$

a. Comparing (1) and (3), we get the value of x

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

b. Comparing (2) and (3), we get the value of y

$$y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

From the above equations, obtain the value of x and y provided $a_1b_2 - a_2b_1 \neq 0$.

12. Equations which are not linear but can be reduced to linear form by some suitable substitutions are called equations reducible to linear form.

Reduced equation can be solved by any of the algebraic method (substitution, elimination or cross multiplication) of solving linear equation.

13. While solving problems based on time, distance and speed; following knowledge may be useful:
If speed of a boat in still water = u km/hr,
Speed of the current = v km/hr
Then,
Speed upstream = (u - v) km/hr
Speed downstream = (u + v) km/hr

MATHS

Polynomials

1. What is a polynomial?

A **polynomial** $p(x)$ in one variable x is an algebraic expression in x of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots\dots + a_2 x^2 + a_1 x + a_0 \text{, where}$$

i. x is a variable

ii. $a_0, a_1, a_2, \dots\dots a_n$ are respectively the coefficients of $x^0, x^1, x^2, x^3, \dots\dots x^n$.

iii. Each of $a_n x^n, a_{n-1} x^{n-1}, a_{n-2} x^{n-2}, \dots\dots a_2 x^2, a_1 x, a_0$, with $a_n \neq 0$, is called the term of a polynomial.

2. The highest exponent of the variable in a polynomial determines the **degree** of the polynomial.

3. Types of polynomials

i. A polynomial of degree zero is called a **constant polynomial**. Examples: $-9x^0$, $\frac{5}{14}$.

ii. A polynomial of degree one is called a **linear polynomial**. It is of the form $ax + b$.
Examples: $x - 2$, $4y + 89$, $3x - z$.

iii. A polynomial of degree two is called a **quadratic polynomial**. It is of the form $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.
Examples: $x^2 - 2x + 5$, $x^2 - 3x$ etc.

iv. A polynomial of degree 3 is called a **cubic polynomial** and has the general form $ax^3 + bx^2 + cx + d$.
For example: $x^3 + 2x^2 - 2x + 5$ etc.

4. Value of the polynomial

If p(x) is a polynomial in x, and k is a real number then the value obtained after replacing x by k in p(x) is called the value of p(x) at x = k which is denoted by p(k).

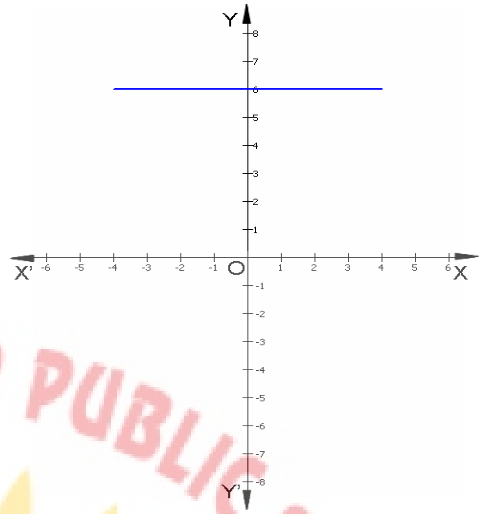
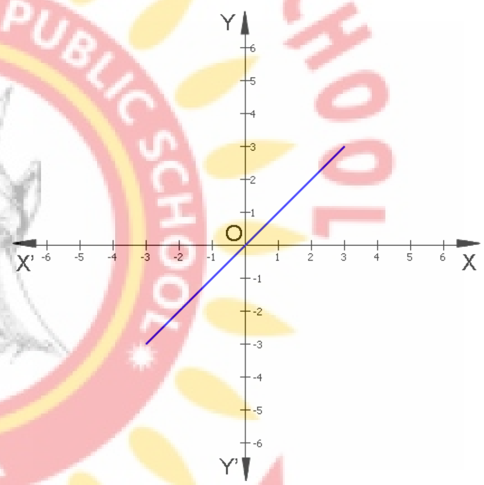
5. Zero of a polynomial

- A real number k is said to be the **zero** of the polynomial $p(x)$, if $p(k) = 0$.
- Zeroes of the polynomial can be obtained by solving the equation $p(x) = 0$.
- It is possible that a polynomial may not have a real zero at all.
- For any linear polynomial $ax + b$, the zero is given by the expression $(-b/a)$ = –(constant term)/(Coefficient of x).

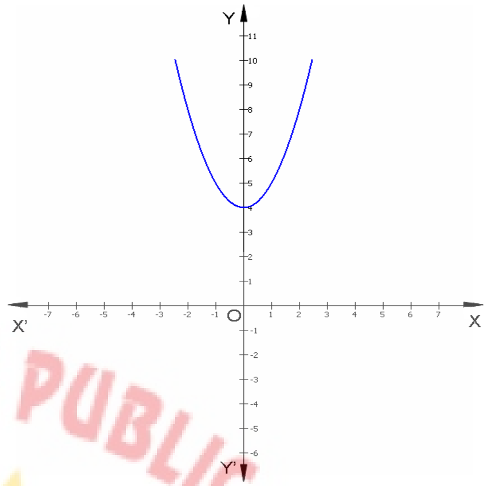
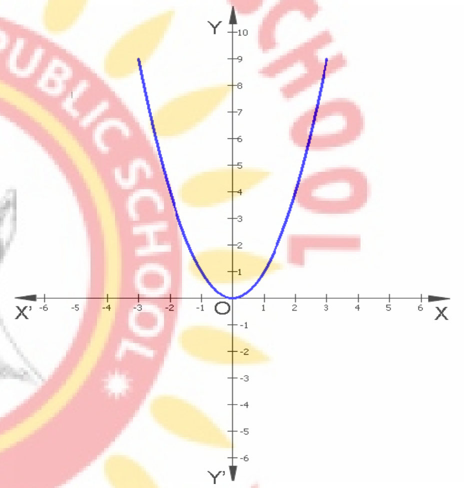
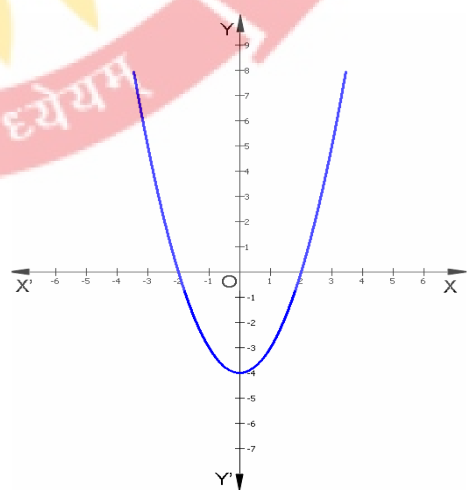
6. Number of zeroes of a polynomial

- The number of real zeros of the polynomial is the number of times its graph touches or intersects x-axis.
- The graph of a polynomial $p(x)$ of degree n intersects or touches the x-axis at at most n points.
- A polynomial of degree n has at most ***n* distinct real zeroes.**

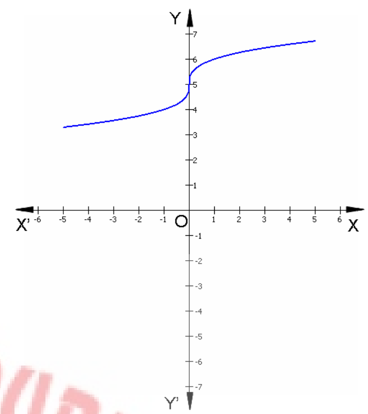
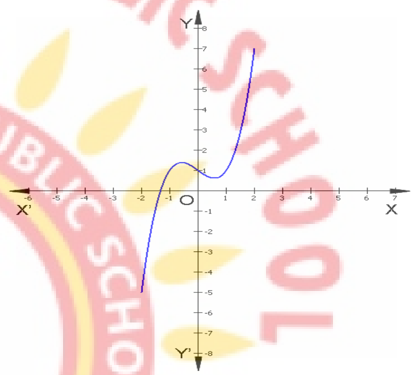
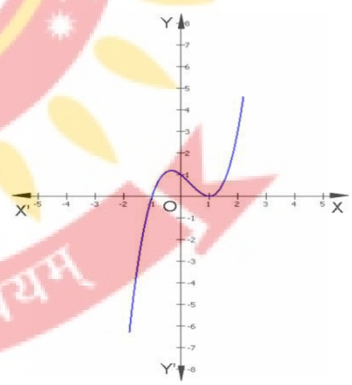
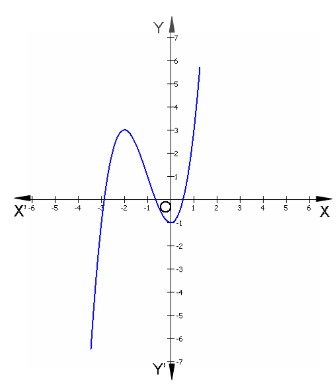
7. A linear polynomial has at most one real zero.

Linear Polynomial having **no zero**.	
Linear Polynomial having **one zero**.	

8. A quadratic polynomial has at most two real zeroes.

Quadratic Polynomial having **no zeroes**.	
Quadratic Polynomial having **one zero**.	
Quadratic Polynomial having **two zeroes**.	

9. A cubic polynomial has at most three real zeroes.

Cubic Polynomial having **no zeroes**.	
Cubic Polynomial having **one zero**.	
Cubic Polynomial having **one zeroes**.	
Cubic Polynomial having **three zeroes**.	

10. **Relationship between zeroes and coefficients** of a polynomial:
    i. For a linear polynomial $ax + b$, $a \neq 0$, the zero is $x = \frac{-b}{a}$. It can be observed that:

$$\frac{-b}{a} = -\frac{\text{constant term}}{\text{Coefficient of } x}$$

    ii. For a **quadratic polynomial** $ax^2 + bx + c$, $a \neq 0$,

Sum of the zeroes $= -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeroes $= \frac{c}{a} = \frac{\text{constant term}}{\text{Coefficient of } x^2}$

    iii. For a **cubic polynomial** $ax^3 + bx^2 + cx + d = 0$, $a \neq 0$,

Sum of zeroes $= \frac{-b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$,

Sum of the product of zeroes taken two at a time $= \frac{c}{a} = \frac{\text{Coefficient of x}}{\text{Coefficient of x}^3}$

Product of zeroes $= -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

11. The quadratic polynomial whose sum of the zeroes = $(\alpha + \beta)$ and product of zeroes = $(\alpha\beta)$ is given by: $\boldsymbol{k\,[x^2 - (\alpha + \beta)\,x + (\alpha\beta)]}$, where k is real.

    If α, β and γ are the zeroes of a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$, then

    $f(x) = k\,(x - \alpha)(x - \beta)(x - \gamma)$

    $f(x) = k\,\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$,

    where k is any non-zero real number.

12. Process of dividing a polynomial $f(x)$ by another polynomial $g(x)$ is as follows:

    Step 1: To obtain the first term of the quotient, divide the highest degree term of the dividend by the highest degree term of the divisor. Then carry out the division process.
    Step 2: To obtain the second term of the quotient, divide the highest degree term of the new dividend by the highest degree term of the divisor. Then again carry out the division process
    Step 3: Continue the process till the degree of the new dividend is less that the degree of the divisor. This will be called the remainder.

13. **Division Algorithm for polynomials:** If $f(x)$ and $g(x)$ are any two polynomials, where $g(x) \neq 0$, then there exists the polynomials $q(x)$ and $r(x)$ such that $f(x) = g(x)\,q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

    So, $q(x)$ is the quotient and $r(x)$ is the remainder obtained when the polynomial $f(x)$ is divided by the polynomial $g(x)$.

14. Factor of the polynomial

If $f(x) = g(x)\,q(x) + r(x)$ and $r(x) = 0$, then polynomial $g(x)$ is a **factor of the polynomial** f(x).

15. Finding zeroes of a polynomial using division algorithm

Division algorithm can also be used to find the **zeroes of a polynomial**. For example, if ‘a’ and ‘b’ are two zeroes of a fourth degree polynomial $f(x)$, then other two zeroes can be found out by dividing $f(x)$ by $(x-a)(x-b)$.

MATHS

$(a+b)^2$

ab+

Arithmetic Progressions

1. What is a Sequence?

- A **sequence** is an arrangement of numbers in a definite order according to some rule.
- The various numbers occurring in a sequence are called its **terms**.
- We denote the terms of a sequence by a_1, a_2, a_3…etc. Here, the subscripts denote the positions of the terms in the sequence.
- In general, the number at the n^{th} place is called the n^{th} term of the sequence and is denoted by a_n. The n^{th} term is also called the **general term** of the sequence.
- A sequence having a finite number of terms is called a **finite sequence**.
- A sequence which do not have a last term and which extends indefinitely is known as an **infinite sequence**.

2. Arithmetic Progression:

- An **arithmetic progression** is a list of numbers in which each term is obtained by adding a fixed number to the preceding term, except the first term.
- Each of the numbers of the sequence is called a **term** of an Arithmetic Progression. The fixed number is called the **common difference**. This common difference could be a positive number, a negative number or even zero.

3. General form and general term (n^{th} term) of an A.P:

- The **general form of an A.P.** is a, a + d, a + 2d, a + 3d….., where 'a' is the first term and 'd' is the common difference.
- The **general term(n^{th} term)** of an A.P is given by $a_n = a + (n - 1)d$, where 'a' is the first term and 'd' is the common difference.

- If the A.P a, a + d, a + 2d, …….., *l* is reversed to *l, l – d, l – 2d,* ….., a then the common difference changes to negative of the common difference of the original sequence.
- To find the n^{th} **term from the end**, we consider this AP backward such that the last term becomes the first term.
  l, $(l - d)$, $(l - 2d)$ ……
  The general term of this AP is given by $a_n = \ell + (n - 1)(-d)$

4. Algorithm to determine whether a sequence is an AP or not:

When we are given an algebraic formula for the general term of the sequence:

Step 1: Obtain a_n.
Step 2: Replace n by (n + 1) in a_n to get a_{n+1}
Step 3: Calculate $a_{n+1} – a_n$
Step 4: Check the value of $a_{n+1} – a_n$.
If $a_{n+1} – a_n$ is independent of n, then the given sequence is an A.P. Otherwise, it is not an A.P.

OR

A list of numbers a_1, a_2, a_3…… is an A.P, if the differences $a_2 – a_1$, $a_3 – a_2$, $a_4 – a_3$ … give the same value, i.e., $a_{k+1} – a_k$ is same for all different values of k.

5. Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

Number of terms	Terms	Common difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

It should be noted that in case of an odd number of terms, the middle term is 'a' and the common difference is 'd' while in case of an even number of terms the middle terms are a – d, a + d and the common differences is 2d.

6. Arithmetic mean:

If three number a, b, c (in order) are in A.P. Then,
b – a = c – b = common difference
⇒ 2b = a + c

Thus a, b and c are in A.P., if and only if 2b = a + c. In this case, b is called the **Arithmetic mean** of a and c.

7. Sum of n terms of an A.P:

- **Sum of n terms of an A.P.** is given by
  $$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
  where 'a' is the first term, 'd' is the common difference and 'n' is the total number of terms.

- **Sum of n terms of an A.P.** is also given by:
  $$S_n = \frac{n}{2}\left[a + \ell\right]$$
  where 'a' is the first term and 'λ' is the last term.

- **Sum of first n natural numbers** is given by $\frac{n(n+1)}{2}$.

8. The n^{th} term of an A.P is the difference of the sum to first n terms and the sum to first (n -1) terms of it. That is, $a_n = S_n - S_{n-1}$

MATHS

Quadratic Equations

1. Introduction to Quadratic equation

If $p(x)$ is a quadratic polynomial, then $p(x) = 0$ is called a **quadratic equation**.
The general or standard form of a quadratic equation, in the variable x, is given by $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

2. Roots of the quadratic equation

- The value of x that satisfies an equation is called the **zeroes** or **roots** of the equation.
- A real number α is said to be a solution/root of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$.
- A quadratic equation has **at most two roots**.

3. A quadratic equation can be solved by following algebraic methods:
   i. Splitting the middle term (factorization)
   ii. Completing squares
   iii. Quadratic formula

4. Splitting the middle term (or factorization) method

- If $ax^2 + bx + c$, $a \neq 0$, can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Steps involved in solving quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ by **splitting the middle term** (or factorization) method:
  Step 1: Find the product ac.
  Step 2: Find the factors of 'ac' that add to up to b, using the following criteria:
  i. If $ac>0$ and $b>0$, then both the factors are positive.
  ii. If $ac>0$ and $b<0$, then both the factors are negative.
  iii. If $ac<0$ and $b>0$, then larger factor is positive and smaller factor is negative.
  iv. If $ac<0$ and $b<0$, then larger factor is negative and smaller factor is positive.
  Step 3: Split the middle term into two parts using the factors obtained in the above step.
  Step 4: Factorize the quadratic equation obtained in the above step by grouping method. Two factors will be obtained.
  Step 5: Equate each of the linear factors to zero to get the value of x.

5. Completing the square method

- Any quadratic equation can be converted to the form $(x + a)^2 – b^2 = 0$ or $(x – a)^2 + b^2 = 0$ by adding and subtracting the constant term. This method of finding the roots of quadratic equation is called the method of completing the square.
- The steps involved in solving a quadratic equation by **completing the square**, are as follows:
  Step 1: Make the coefficient of x^2 unity.
  Step 2: Express the coefficient of x in the form $2 \times x \times p$.
  Step 3: Add and subtract the square of p.
  Step 4: Use the square identity $(a + b)^2$ or $(a – b)^2$ to obtain the quadratic equation in the required form $(x + a)^2 - b^2 = 0$ or $(x – a)^2 + b^2 = 0$.
  Step 5: Take the constant term to the other side of the equation.
  Step 6: Take the square root on both the sides of the obtained equation to get the roots of the given quadratic equation.

6. Quadratic formula

The roots of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ can be calculated by using the **quadratic formula:**

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad \frac{-b-\sqrt{b^2-4ac}}{2a}, \text{ where } b^2-4ac \geq 0$$

If $b^2 - 4ac < 0$, then equation does not have real roots.

7. Discriminant of a quadratic equation

For the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, the expression $b^2 - 4ac$ is known as **discriminant**.

8. **Nature of the roots** of a quadratic equation:

i. If $b^2 - 4ac > 0$, the quadratic equation has **two distinct real roots**.
ii. If $b^2 - 4ac = 0$, the quadratic equation has **two equal real roots**.
iii. If $b^2 - 4ac < 0$, the quadratic equation has **no real roots**.

9. There are many equations which are not in the quadratic form but can be reduced to the quadratic form by simplifications.

10. Application of quadratic equations

- The applications of quadratic equation can be utilized in solving real life problems.
- Following points can be helpful in solving word problems:
  i. Every two digit number 'xy' where x is a ten's place and y is a unit's place can be expressed as $xy = 10x + y$.
  ii. Downstream: It means that the boat is running in the direction of the stream
  Upstream: It means that the boat is running in the opposite direction of the stream
  Thus, if
  Speed of boat in still water is x km/h
  And the speed of stream is y km/h
  Then the speed of boat downstream will be $(x + y)$ km/h and in upstream it will be $(x - y)$ km/h.
  iii. If a person takes x days to finish a work, then his one day's work = $\frac{1}{x}$

MATHS

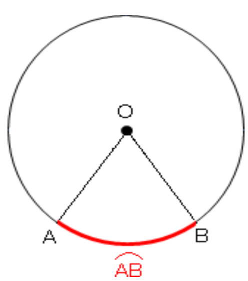
Areas Related to Circles

1. A **circle** is a set of points in a plane that are at an equal distance from a fixed point. The fixed point is called the centre of circle and equal distance is called the radius of the circle.
2. A line segment joining the centre of the circle to a point on the circle is called its **radius**.
3. A line segment joining any two points of a circle is called a **chord**. A chord passing through the centre
4. of circle is called its **diameter**.
5. The distance around the boundary of the circle is called **the perimeter or the circumference** of the circle.
6. Circumference (perimeter) of a circle = πd or $2\pi r$, where d is he diameter, r is the radius of the circle and $\pi = \frac{22}{7}$.
7. Perimeter of a semi circle or protractor = $\pi r + 2r$
8. Perimeter of a quadrant = $\frac{1}{4}$ Circumference $+ 2r$ = $\frac{\pi r}{2} + 2r$
9. Distance moved by a wheel in 1 revolution = Circumference of the wheel.

   Number of revolutions in one minute = $\frac{\text{Distance moved in 1 minute}}{\text{Circumference}}$
10. The region enclosed inside a circle is called its **area.**
11. Area of a circle = πr^2
12. Area of a semi circle = $\frac{1}{2}\pi r^2$
13. Area of a quadrant = $\frac{1}{4}$ Area of circle $= \frac{1}{4}\pi r^2$
14. Circles having the same centre but different radii are called **concentric circles**.

    Area enclosed by two concentric circles = $\pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right) = \pi(R+r)(R-r)$

    Where, R and r are radii of two concentric circles
15. The part of the circumference between the two end points of the chord is called an **arc**. In the figure, arc $\overset{\frown}{AB}$ is shown.



16. A diameter of circle divides a circle into two equal arcs, each known as a **semi-circle**.

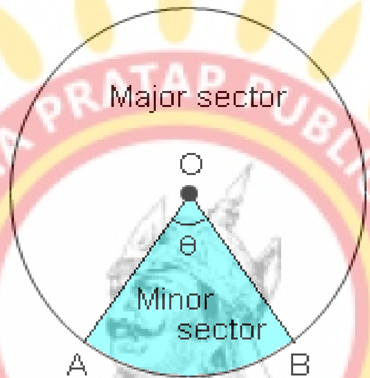
17. An arc of a circle whose length is less than that of a semicircle of the same circle is called a **minor arc**.

18. An arc of a circle whose length is greater than that of a semicircle of the same circle is called a **major arc**.

19. Length of an arc = $\frac{\pi r\theta}{180^\circ}$

20. The region bounded by an arc of a circle and two radii at its end points is called a **sector**.

If the central angle of a sector is more than 180°, then the sector is called a **major sector** and if the central angle is less than 180°, then the sector is called a **minor sector**.



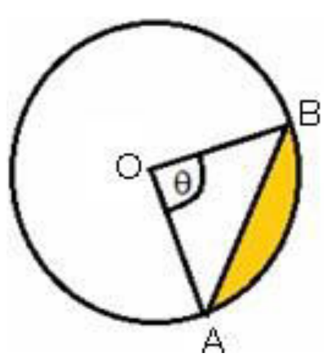
21. Perimeter of sector of angle θ = $\frac{\pi r\theta}{180^\circ} + 2r$

22. Area of a sector of angle θ = $\frac{\pi r^2\theta}{360^\circ}$

23. Area of major sector = πr^2 – Area of minor sector

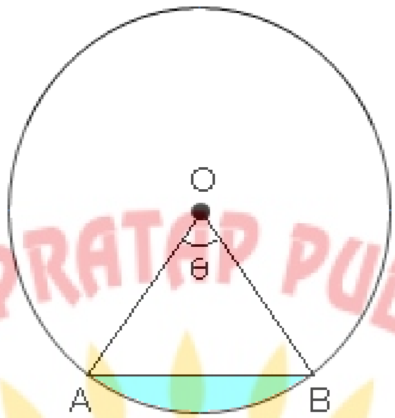
24. A chord divides the interior of a circle into two parts, each called a segment.

The segment which is smaller than the portion of semi-circle is called the **minor segment** and the segment which is larger than the portion of semi-circle is called the **major segment**. In the circle shown, the yellow portion is the minor segment while the non-shaded portion is the major segment.



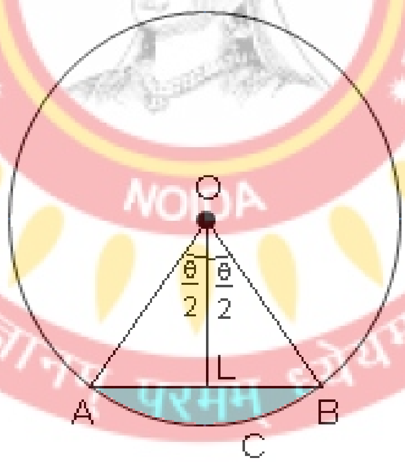
25. Perimeter of segment of angle $\theta = \frac{\pi r\theta}{180^\circ} + \overline{AB}$

26. Area of minor segment = Area of sector - Area of Δ ABC



27. Area of minor segment can also be written as:

Area of segment $ACB = \left\{\frac{\pi}{360^\circ}\times\theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$



28. Area of major segment = Area of the circle – Area of minor segment

MATHS

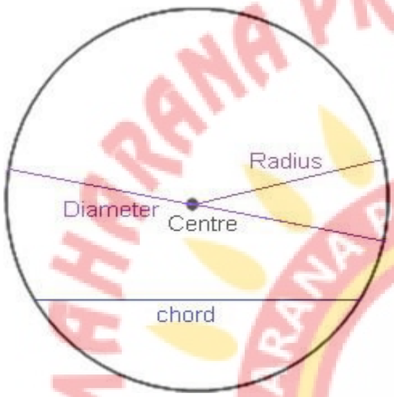
Circles

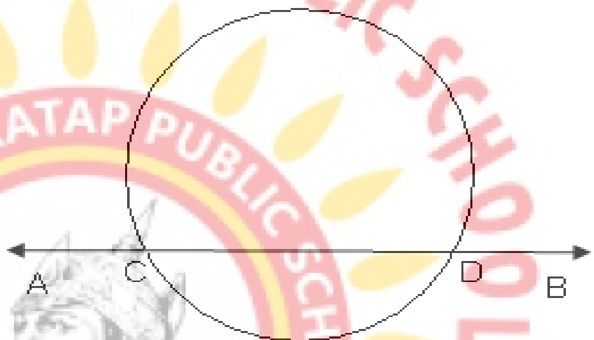
1. Introduction to Circle

A **circle** is the locus of a point which lies in the plane in such a manner that its distance from a fixed point in the plane is constant. The fixed point is called the **centre** and the constant distance is called the **radius** of the circle.

2. Parts of the circle

- A line segment that joins any two points lying on a circle is called the **chord** of the circle.
- A chord passing through the centre of the circle is called **diameter** of the circle.
- A line segment joining the centre and a point on the circle is called **radius** of the circle.
- A line which intersects a circle at two distinct points is called a **secant** of the circle.

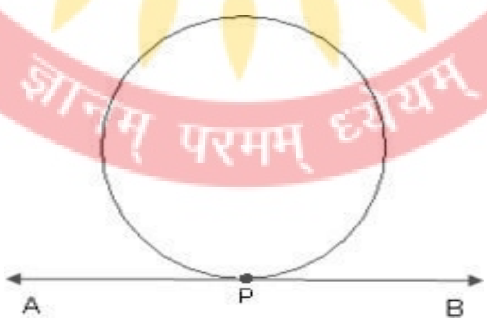




In the figure, AB is a secant to the circle.

3. Tangent to the circle

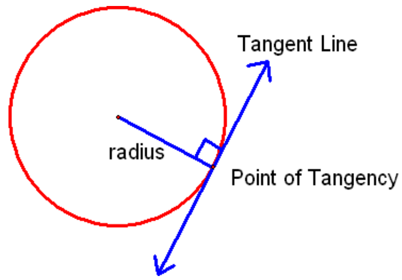
A **tangent** to the circle is a line that intersects the circle (touches the circle) at only one point. The word 'tangent' comes from the Latin word 'tangere', which means to touch. The common point of the circle and the tangent is called **point of contact**.



In the figure, AB is a tangent to the circle and P is the point of contact.

4. Important facts about tangent

- The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact. This point of contact is also called as point of tangency.



- A line drawn through the end of a radius (point on circumference) and perpendicular to it is a tangent to the circle.

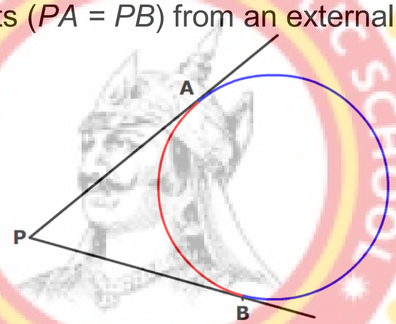
5. Number of tangents on a circle

- There is no tangent possible to a circle from the point (or passing through a point) lying inside the circle.
- There are **exactly two tangents** possible to a circle **through a point outside the circle**.
- At any point on the circle, there can be one and only one tangent possible.

6. Length of the tangent

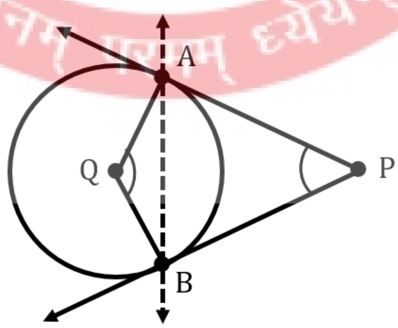
The length of the segment of the tangent from the external point P and the point of contact with the circle is called the **length of the tangent**.

- The lengths of tangents drawn from an external point to the circle are equal.
- The figure shows two equal tangents (*PA* = *PB*) from an external point *P*.



7. Angle between two tangents from an external point

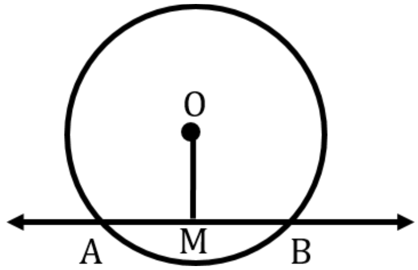
- The centre of a circle lies on the bisector of the angle between the two tangents drawn from an external point.
- Angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.



In the figure, angle P and angle Q are supplementary.

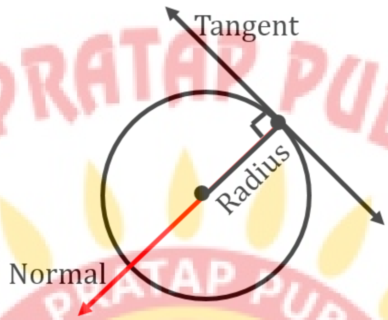
8. Perpendicular from the centre

Perpendicular drawn from the centre to any chord of the circle, divides it into two equal parts.
In the figure, OM is perpendicular to AB and AM = MB.



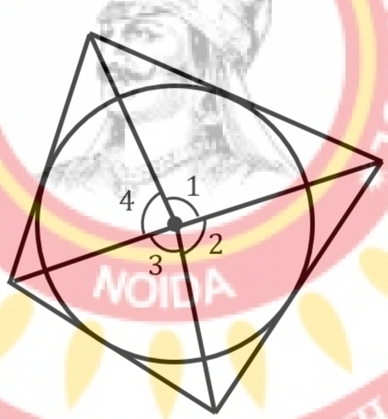
9. Normal to the circle

The line containing the radius through the point of contact is called the normal to the circle at that point.



10. Inscribed circle

Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



In the figure, angles 1 and 3 are supplementary. Accordingly, angles 2 and 4 are supplementary.

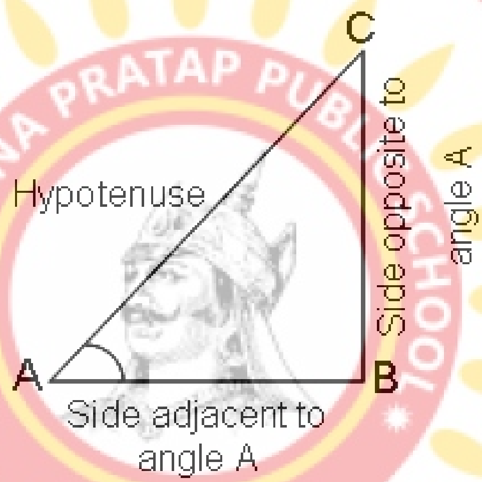
MATHS

Introduction to Trigonometry

1. **Meaning (Definition) of Trigonometry**
   The word trigonometry is derived from the Greek words '*tri*' meaning three, '*gon*' meaning sides and 'metron' meaning measure.
   Trigonometry is the study of relationships between the sides and the angles of the triangle.

2. **Positive and negative angles**
   Angle measured in anticlockwise direction is taken as positive angle whereas the angle measured in clockwise direction is taken as negative angle.

3. **Trigonometric Ratios**
   Ratio of the sides of a right triangle with respect to the acute angles is called the **trigonometric ratios** of the angle.
   Trigonometric ratios of the acute angle A in right triangle ABC are given as follows:



i. $\sin\angle A = \dfrac{\text{side opposite to}\angle A}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{p}{h}$

ii. $\cos\angle A = \dfrac{\text{side adjacent to}\angle A}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{b}{h}$

iii. $\tan\angle A = \dfrac{\text{side opposite to}\angle A}{\text{side adjacent to}\angle A} = \dfrac{BC}{AB} = \dfrac{p}{b}$

iv. $\text{cosec}\ \angle A = \dfrac{\text{hypotenuse}}{\text{side opposite to}\angle A} = \dfrac{AC}{BC} = \dfrac{h}{p}$

v. $\sec\angle A = \dfrac{\text{hypotenuse}}{\text{side adjacent to}\angle A} = \dfrac{AC}{AB} = \dfrac{h}{b}$

vi. $\cot\angle A = \dfrac{\text{side adjacent to}\angle A}{\text{side opposite to}\angle A} = \dfrac{AB}{BC} = \dfrac{b}{p}$

4. **Important facts about Trigonometric ratios**
   - Trigonometric ratios of an acute angle in a right triangle represents the relation between the angle and the sides.
   - The ratios defined above can be rewritten as sin A, cos A, tan A, cosec A, sec A and cot A.

- Each trigonometric ratio is a real number and it has not unit.

- All the trigonometric symbols i.e., cosine, sine, tangent, cotangent, secant and cosecant, have no literal meaning.
- $(\sin\theta)^n$ is generally written as $\sin^n \theta$, n being a positive integer. Similarly, other trigonometric ratios can also be written.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

5. Pythagoras theorem:
It states that "in a right triangle, square of the hypotenuse is equal to the sum of the squar es of the other two sides".
Pythagoras theorem can be used to obtain the length of the side of a right angled triangle when the other two sides are already given.

6. **Relation between trigonometric ratios**:
The ratios cosec A, sec A and cot A are the reciprocals of the ratios sin A, cos A and tan A respectively as given:

i. $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

ii. $\sec\theta = \frac{1}{\cos\theta}$

iii. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

iv. $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

7. **Values of Trigonometric ratios** of some specific angles:

$\angle$A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

- The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than 1 or equal to 1.
- The value of $\sin\theta$ increases from 0 to 1 when θ increases from 0^o to 90^o.
- The value of cos θ decreases from 1 to 0 when θ increases from 0^o to 90^o.
- If one of the sides and any other parts like either an acute angle or any side of a right triangle are known, the remaining sides and angles of the triangle can be obtained using trigonometric ratios.

8. **Trigonometric ratios of complementary angles**:

Two angles are said to complementary angles if their sum is equal to 90^o. Based on this relation, the trigonometric ratios of complementary angles are given as follows:

i. $\sin (90^o - A) = \cos A$
ii. $\cos (90^o - A) = \sin A$
iii. $\tan (90^o - A) = \cot A$
iv. $\cot (90^o - A) = \tan A$
v. $\sec (90^o - A) = \operatorname{cosec} A$
vi. $\operatorname{cosec} (90^o - A) = \sec A$

Note: $\tan 0^o = 0 = \cot 90^o$, $\sec 0^o = 1 = \operatorname{cosec} 90^o$, $\sec 90^o$, $\operatorname{cosec} 0^o$, $\tan 90^o$ and $\cot 0^o$ are not defined.

9. **Definition of Trigonometric Identity**

An equation involving trigonometric ratios of an angle, say θ, is termed as a **trigonometric identity** if it is satisfied by all values of θ.

10. **Basic trigonometric identities**

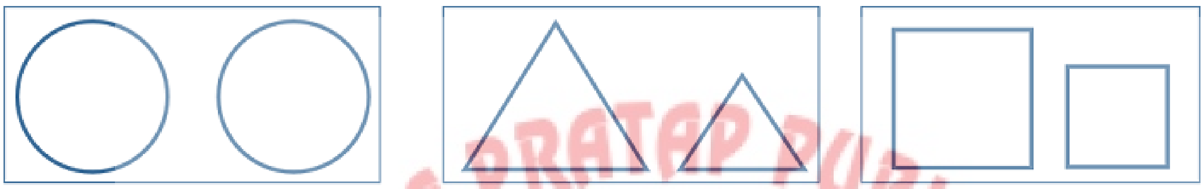
i. $\sin^2 \theta + \cos^2 \theta = 1$
ii. $1 + \tan^2 \theta = \sec^2 \theta$; $0 \le \theta < 90$
iii. $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$; $0 \le \theta < 90$

MATHS

Triangles

1. **Congruent figures:**
   Two geometrical figures are called **congruent** if they superpose exactly on each other, that is, they are of the same shape and size.

2. **Similar figures:**
   Two figures are **similar**, if they are of the same shape but not necessarily of the same size.



Group 1 Group 2 Group 3

3. All congruent figures are similar but the similar figures need not to be congruent.

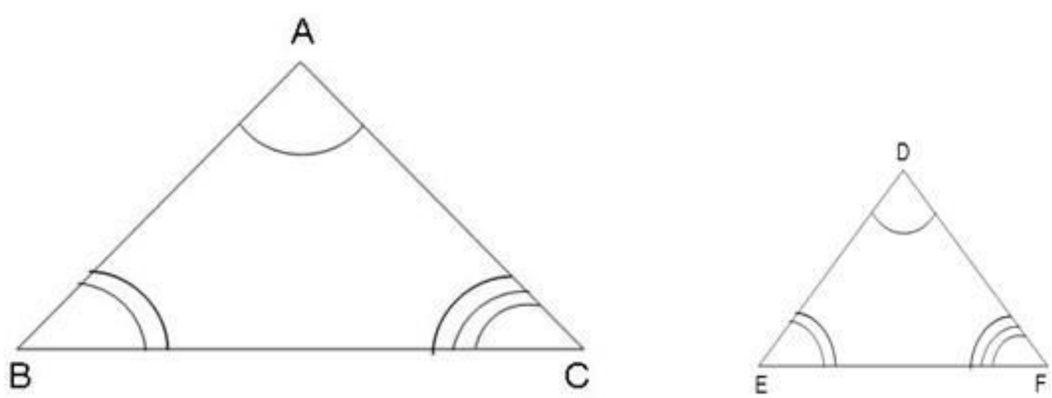
4. Two **polygons** having the same number of sides are **similar** if
   i. their corresponding angles are equal and
   ii. their corresponding sides are in the same ratio (or proportion).
   Note: Same ratio of the corresponding sides means the **scale factor** for the polygons.

5. Important facts related to similar figures are:
   i. All circles are similar.
   ii. All squares are similar.
   iii. All equilateral triangles are similar.
   iv. The ratio of any two corresponding sides in two equiangular triangles is always same.

6. Two **triangles are similar** (~) if
   i. Their corresponding angles are equal.
   ii. Their corresponding sides are in same ratio.

7. If the angles in two triangles are:
   i. Different, the triangles are neither similar nor congruent.
   ii. Same, the triangles are similar.
   iii. Same and the corresponding sides are of the same size, the triangles are congruent.

   In the given figure, $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$, which means triangles ABC and DEF are similar which is represented by $\Delta ABC \sim \Delta DEF$



8. If $\Delta ABC \sim \Delta PQR$, then
   i. $\angle A = \angle P$
   ii. $\angle B = \angle Q$
   iii. $\angle C = \angle R$
   iv. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

9. Equiangular triangles:
Two triangles are **equiangular** if their corresponding angles are equal. The ratio of any two corresponding sides in such triangles is always the same.

10. **Basic Proportionality Theorem** (Thales Theorem):
If a line is drawn parallel to one side of a triangle to intersect other two sides in distinct points, the other two sides are divided in the same ratio.

11. Converse of BPT:
If a line divides any two sides of a triangle in the same ratio then the line is parallel to the third side.

12. A line drawn through the mid-point of one side of a triangle which is parallel to another side bisects the third side. In other words, the line joining the mid-points of any two sides of a triangle is parallel to the third side.

13. AAA (Angle-Angle-Angle) similarity criterion:
If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

14. AA (Angle-Angle) similarity criterion:
If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal.
Thus, **AAA similarity criterion** changes to **AA similarity criterion** which can be stated as follows:
If two angles of one triangle are respectively equal to two angles of other triangle, then the two triangles are similar.

15. Converse of *AAA* similarity criterion:
If two triangles are similar, then their corresponding angles are equal.

16. SSS (Side-Side-Side) similarity criterion:
If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

17. Converse of SSS similarity criterion:
If two triangles are similar, then their corresponding sides are in constant proportion.

18. SAS (Side-Angle-Side) similarity criterion:
If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

19. Converse of SAS similarity criterion:
If two triangles are similar, then one of the angles of one triangle is equal to the corresponding angle of the other triangle and the sides including these angles are in constant proportion.

20. RHS (Right angle-Hypotenuse-Side) criterion:
If in two right triangles, hypotenuse and one side of one triangle are proportional to the hypotenuse and one side of another triangle, then the two triangles are similar. This criteria is referred as the RHS similarity criterion

21. Pythagoras Theorem:
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
Thus, in triangle ABC right angled at B, $AB^2 + BC^2 = AC^2$

22. Converse of Pythagoras Theorem:
If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

23. The **ratio of the areas of two similar triangles** is equal to the square of the ratio of their corresponding sides.
Thus, if $\Delta ABC \sim \Delta PQR$, then $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$

Also, the ration of the areas of two similar triangles is equal to the ration of the squares of the corresponding medians.

24. Some important results of similarity are:
- In an equilateral or an isosceles triangle, the altitude divides the base into two equal parts.
- If a perpendicular is drawn from the vertex of the right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.
- The area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonal.
- Sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.
- In an equilateral triangle, three times the square of one side is equal to four times the square of one of its altitudes.

MATHS

Statistics

1. Three measures of central tendency are:

   i. Mean ii. Median iii. Mode

2. The **arithmetic mean**, also called the average, is the quantity obtained by adding all the observations and then dividing by the total number of observations.

3. Arithmetic mean may be computed by anyone of the following methods:
   i. Direct method
   ii. Short-cut method/ Assumed mean method
   iii. Step-deviation method

4. **Direct method** of finding mean:
   If a variant X takes values x_1, x_2, x_3.... x_n with corresponding frequencies f_1, f_2, f_3 ,... f_n respectively, then arithmetic mean of these values is given by:

$$\overline{X} = \frac{\sum_{i=1}^{n} f_i x_i}{N}, \text{ where } N = \sum_{i=1}^{n} f_1 + f_1 + f_2 ... + f_n$$

5. **Class mark** = ½ (Upper class limit + Lower class limit)

6. **Short-cut method/ assumed mean method** of finding mean:
   Let $x_1, x_2 ...., x_n$ be values of a variable X with corresponding frequencies $f_1, f_2, f_3 ..... , f_n$ respectively. Let A be the assumed mean, $d_i = (x_i - A)$. Then:

$$\overline{X} = A + \frac{1}{N}\left(\sum_{i=1}^{n} f_i d_i\right)$$

   Note that in case of continuous frequency distribution, the values of x_1, x_2, x_3 ..., x_n are taken as the mid-points or class-marks of the various classes.

7. **Step-deviation method** of finding mean:
   Let $x_1, x_2 ...., x_n$ be values of a variable X with corresponding frequencies $f_1, f_2, f_3 ..... , f_n$ respectively. Let A be the assumed mean. Then:

$$\overline{X} = A + h\left\{\frac{1}{N}\sum_{i=1}^{n} f_i u_i\right\}$$

   Here, h is generally taken as common factor of the deviations, in case of ungrouped frequency distribution. And, in case of grouped frequency distribution, h is the class width, $u_i = \frac{x_i - A}{h} = \frac{d_i}{h}$

   Note that in case of continuous frequency distribution, the values of x_1, x_2, x_3 ..., x_n are taken as the mid-points or class-marks of the various classes.

8. The step deviation method will be convenient to apply if all the deviations (*d's*) have a common factor.

9. If class mark obtained, are in decimal form, then step deviation method is preferred to calculate mean.

10. **Median** is a measure of central tendency which gives the value of the middle observation in the data, arranged in order. It is that value such that the number of observations above it is equal to the number of observations below it.

11. For finding the median of a raw data, we arrange the given data in increasing or decreasing order.

    If n is odd, then median is the value of $\left(\frac{n+1}{2}\right)^{th}$ observation.

    If n is even, then median is the arithmetic mean of the values of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

12. The **cumulative frequency** of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class to the frequency of the class.

13. In case of an **ungrouped frequency distribution**, we calculate the **median** by following the steps given below:

    Step 1: Find the cumulative frequencies (c.f.) and obtain $N = \sum f_1$.

    Step 2: Find $\frac{N}{2}$.

    Step 3: Look for the cumulative frequency (c. f.) just greater than $\frac{N}{2}$ and determine the corresponding value of the variable. The value so obtained is the median.

14. In case of a **continuous frequency distribution**, we calculate the **median** by following the steps:

    Step 1: Find the cumulative frequencies (c.f.) and obtain $N = \sum f_1$.

    Step 2: Find $\frac{N}{2}$.

    Step 3: Look for the cumulative frequency (c. f.) just greater than $\frac{N}{2}$ and determine the corresponding class. This class is known as the median class. (Note that the value of the median will lie in this class)

    Step 4: Use the following formula to find median:

    $$\text{Median} = l + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

    Here, l = lower limit of the median class
    f = frequency of the median class
    h = width (size) of the median class
    cf = cumulative frequency of the class preceding the median class
    $N = \sum f_1$.

15. **Mode** is the value of the most frequently occurring observation in the data.

16. In an ungrouped frequency distribution, mode is the value of the variable having maximum frequency.

17. In a **grouped frequency distribution**, the modal class is the one with highest frequency and the **mode** can be calculated by the following formula

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

l = lower limit of the modal class
h = size of the class interval
f_1 = frequency of the modal class
f_0 =frequency of the class preceding the modal class
f_2 = frequency of the class succeeding the modal class

18. The most frequently used measure of central tendency is the mean, because the mean is calculated by taking into account all the observations of a given data. And it lies between the smallest and the largest value of the data.

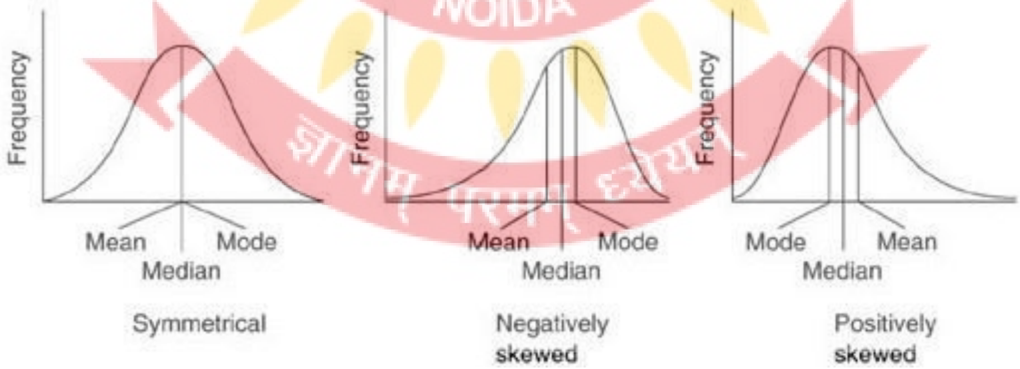
19. The biggest drawback in considering mean is that it is affected by the extreme values. One large or small number can distort the average. In that case, median is a better measure of central tendency. While, when the most repeated value or the most wanted one is required, then mode is used.

20. When all three measures of central tendency are equal, the distribution is called **symmetrical distribution**.

21. When the values of mean, median and mode are not equal, then the distribution is known as **asymmetrical or skewed**. In this case, the distribution can be positively skewed or negatively skewed.

    Negatively skewed distributions have a few extremely low scores, while positively skewed distributions have a few extremely high scores.

    i. When the data is negatively skewed, then Mean < Median < Mode
    ii. When the positively skewed, then Mean > Median > Mode



22. Three measure of central values are connected by the following relation:

    3 Median = Mode + 2 Mean

23. The **cumulative frequency** is the accumulated or sum of frequencies up to a particular point. A table showing the cumulative frequencies is called a **cumulative frequency distribution**.

24. There are two types of cumulative frequencies:
    i. Less than type cumulative frequency distribution: It is found by adding sequentially the frequencies of all the earlier classes including the class adjacent to which it is written. The cumulate is started from the lowest to the highest size.
    ii. More than type cumulative frequency distribution: It is obtained by finding the cumulate of frequencies starting from the highest to the lowest class.

25. A cumulative frequency distribution can be represented graphically by means of an **ogive.**

26. There are two types of ogives:
    i. **'Less than' ogive**: In a less than ogive the upper limit of a class (x axis) is plotted against its cumulative frequency (y axis) as a point on the ogive. The 'less than ogive' is a rising curve.
    ii. **'More than' ogive**: In a 'more than ogive' the lower limit of a class (x axis) is plotted against its cumulative frequency (y axis) as a point on the ogive. The 'more than ogive' is a falling curve.

27. The ogives can be drawn only when the given class intervals are continuous and if this is not the case then first the class intervals are made continuous.

28. In order to determine the **median from less than ogive or more than ogive**, we follow the steps given below:

    Step 1: Draw more than or less than ogive as asked in question. Find $\frac{N}{2}$, where N is the total number of observations.

    Step 2: Locate the $\frac{N}{2}$ cumulative frequency on the y-axis.

    Step 3: Draw a line parallel to x-axis through the point obtained in step 2, cutting the cumulative frequency curve at a point *P* (say).

    Step 4: Draw perpendicular *PM* from *P* on the *x*-axis. The *x*-coordinate of point M is the median value.

29. If we draw **less than ogive and more than ogive on the same graph**, then **median** can be obtained by following the steps given below:

    Step 1: Draw both ogives on the same graph.
    Step 2: Identify the point of intersection of both ogives and mark it as *Q* (say).
    Step 3: Draw perpendicular from Q on x-axis.
    Step 4: The point of perpendicular on x-axis is the median.

MATHS

Surface Areas and Volumes

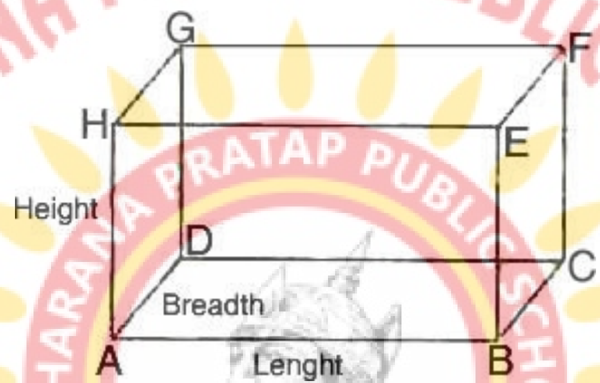
1. **Surface area** of a solid is the sum of the areas of all its faces.
2. The space occupied by a solid object is the **volume** of that object.
3. If l, b, h denote respectively the length, breadth and height of a **cuboid**, then:

   Lateral surface area or Area of four walls = $2(\ell + b)h$

   Total surface area = $2(\ell b + bh + h\ell)$

   Volume = $\ell \times b \times h$

   Diagonal of a cuboid = $\sqrt{\ell^2 + b^2 + h^2}$



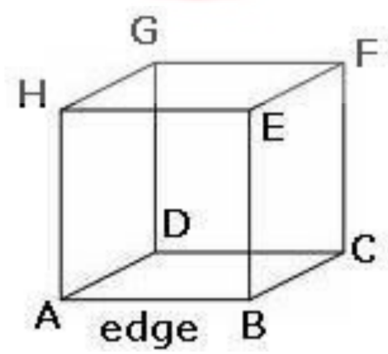
4. If the length of each edge of a **cube** is 'a' units, then:

   Lateral surface area = 4 x $(\text{edge})^2$

   Total surface area = 6 x $(\text{edge})^2$

   Volume = $(\text{edge})^3$

   Diagonal of a cube = $\sqrt{3}$ x edge



5. If *r* and *h* respectively denote the radius of the base and the height of a **right circular cylinder**, then:

Area of each end or Base area = πr^2

Area of curved surface or lateral surface area = perimeter of the base x height = $2\pi rh$

Total surface area (including both ends) = $2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

Volume = Area of the base x height = $\pi r^2 h$

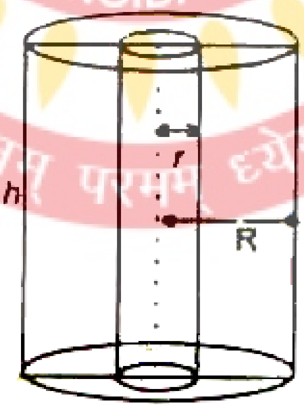
6. If *R* and *r* respectively denote the external and internal radii of a **right circular hollow cylinder** and *h* denotes its height, then:

Area of each end = $\pi R^2 - \pi r^2$

Area of curved surface = $2\pi(R + r)h$

Total surface area = (Area of curved surface) + 2(Area of each end)

$= 2\pi(R + r)h + 2\,(\pi R^2 - \pi r^2)$



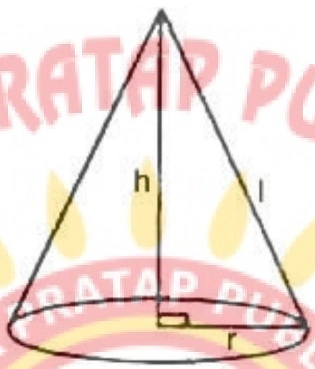
7. If r, h and l respectively denote the radius, height and slant height of a **right circular cone**, then:

   Slant height (ℓ) = $\sqrt{h^2 + r^2}$

   Area of curved surface = $\pi r\ell = \pi r\sqrt{h^2 + r^2}$

   Total surface area = Area of curved surface + Area of base = $\pi r\ell + \pi r^2 = \pi r(\ell + r)$

   Volume = $\frac{1}{3}\pi r^2 h$



8. If r is the radius of a **sphere**, then:

   Surface area = $4\pi r^2$

   Volume = $\frac{4}{3}\pi r^3$

9. If r is the radius of a **hemisphere**, then:

   Area of curved surface = $2\pi r^2$

   Total surface Area = Area of curved surface + Area of base

   $= 2\pi r^2 + \pi r^2$

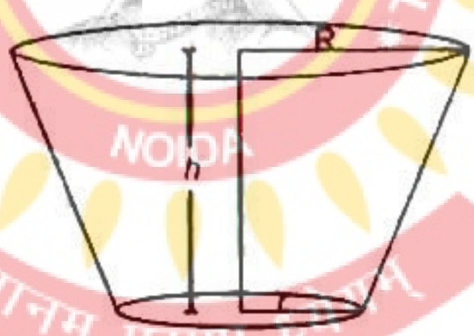
   $= 3\pi r^2$

   Volume = $\frac{2}{3}\pi r^3$

10. The total surface area of the solid formed by the combination of solids is the sum of the curved surface area of each of the individual solids.

11. The volume of the solid formed by the combination of basic solids is the sum of the volumes of each of the basic solids.

12. If a right circular cone is cut off by a plane parallel to its base, then the portion of the cone between the plane and the base of the cone is called a **frustum of the cone.**



13. If *h* is the height, *l* is the slant height, *R* and *r* are the radii of the upper and lower ends of a **frustum of a cone**, then:

    Curved surface area = $\pi (R + r) \ell$

    Total surface area = $\pi (R + r) \ell + \pi[R^2 + r^2]$

    Volume = $\frac{1}{3}\pi h \left[R^2 + r^2 + Rr\right]$

14. When a solid is melted and converted to another, volume of both the solids remains the same, assuming there is no wastage in the conversions. However, the surface area of the two solids may or may not be the same.

15. The solids having the same curved surface do not necessarily occupy the same volume and vice versa.

MATHS

Real Numbers

1. Euclid's Division Lemma:

Given positive integers a and *b,* there exists unique integers q and r satisfying
$a = bq + r$, where $0 \le r < b$

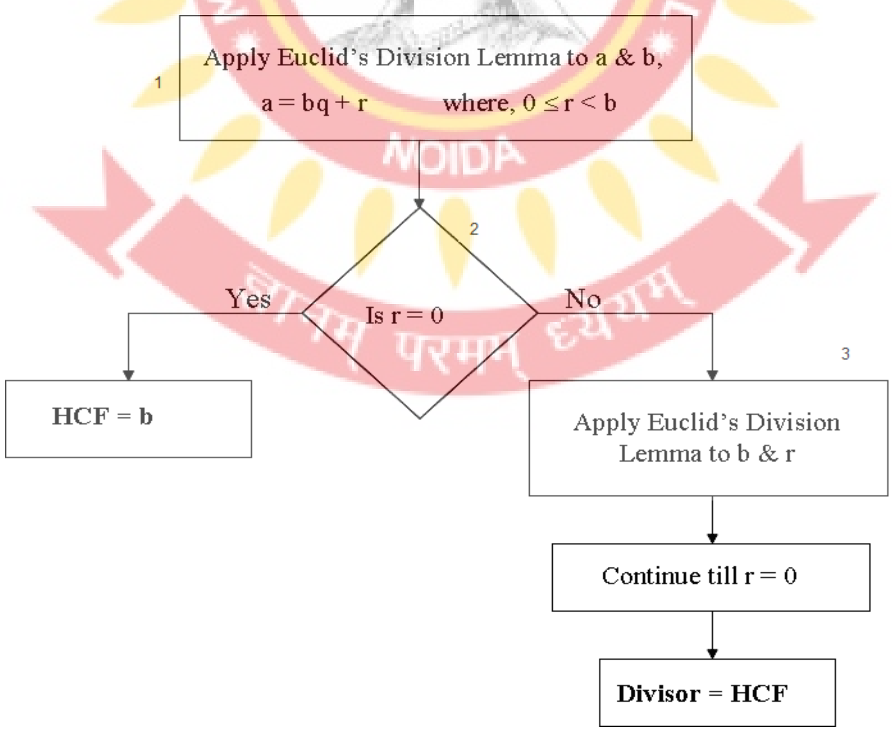
- **Lemma** is a proven statement used for proving another statement.

2. Euclid's Division Algorithm:

- An **algorithm** is a series of well defined steps which gives a procedure for solving a type of problem.
- This algorithm is a technique to compute the **H.C.F** of two given positive integers.
- According to this algorithm, the **HCF** of any two positive integers 'a' and 'b', with a > b, is obtained by following the steps given below:

Step 1: Apply Euclid's division lemma, to 'a' and 'b', to find q and r, such that $a = bq + r$, $0 \le r < b$.
Step 2: If $r = 0$, the HCF is b. If $r \neq 0$, apply Euclid's division lemma to b and r.
Step 3: Continue the process till the remainder is zero. The divisor at this stage will be HCF (a, b).
Also, note that HCF (a, b) = HCF (b, r).

Euclid's Division Algorithm can be summarized as follows:



- Euclid's Division Algorithm is stated for only positive integers but it can be extended for all integers except zero, i.e., $b \neq 0$.

3. **Real Numbers:**

- The numbers which can be represented in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called **Rational numbers**.
- Any number that cannot be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called **Irrational numbers**.
- There are more irrational numbers than rational numbers between two consecutive numbers.
- Rational and Irrational numbers together constitute **Real numbers**.

4. **Properties of Irrational numbers**:

i. The **Sum, Difference, Product** and **Division** of two irrational numbers need not always be an irrational number.

ii. **Negative** of an irrational number is an irrational number.

iii. **Sum** of a **rational** and an **irrational** number is irrational.

iv. **Product** and **Division** of a non-zero rational and irrational number is always irrational.

5. **Fractions:**

- **Terminating fractions** are the fractions which leaves remainder 0 on normal division.
- **Recurring fractions** are the fractions which never leave a remainder 0 on normal division.

6. **Properties related to prime numbers:**

- If p is a prime and divides a^2, then p divides a, where 'a' is a positive integer.
- If p is a prime, then $\sqrt{p}$ is an irrational number.
- A number ends with the digit zero if and only if it has 2 and 5 as two of its prime factors.

7. **Decimal Expansion:**

- The decimal expansion of rational number is either **terminating** or **non-terminating recurring (repeating).**
- If the decimal expansion of rational number **terminates**, then we can express the number in the form of $\frac{p}{q}$, where p and q are co prime, and the prime factorization of **q is of the form 2^n5^m**, where n and m are non negative integers.
- If x = $\frac{p}{q}$ is a rational number, such that the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which **terminates.**

- If the denominator of a rational number is of the form 2^n5^m, then it will terminate after n places if n > m or after m places if m > n.

- The decimal expansion of an irrational number is **non-terminating, non-recurring.**

8. **Fundamental Theorem of Arithmetic**:

Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

- The procedure of finding **HCF(Highest Common Factor)** and **LCM(Lowest Common Multiple)** of given two positive integers a and b:

i. Find the prime factorization of given numbers.

ii. HCF(a, b) = **Product of the smallest power of each common prime factors in the numbers.**

iii. LCM(a, b) = **Product of the greatest power of each prime factors, involved in the numbers.**

9. **Relationship between HCF and LCM** of two numbers:

If a and b are two positive integers, then HCF (a, b) $\times$ LCM (a, b) = a $\times$ b

10. **Relationship between HCF and LCM of three numbers:**

$$LCM\ (p, q, r) = \frac{p.\ q.\ r\ .\ \text{HCF}\ (p, q, r)}{\text{HCF}\ (p, q).\text{HCF}\ (q, r).\ \text{HCF}\ (p, r)}$$

$$HCF\ (p, q, r) = \frac{p.\ q.\ r\ .\ \text{LCM}\ (p, q, r)}{\text{LCM}\ (p, q).\ \text{LCM}\ (q, r).\ \text{LCM}\ (p, r)}$$

MATHS

$(a+b)^2$

ab+

Probability

1. **Probability** is a quantitative measure of uncertainty.
2. In the **experimental approach** to probability, we find the probability of the occurrence of an event by actually performing the experiment a number of times and adequate recording of the happening of event.
3. In the **theoretical approach** to probability, we try to predict what will happen without actually performing the experiment.
4. The experimental probability of an event approaches to its theoretical probability if the number of trials of an experiment is very large.
5. An **outcome** is a result of a single trial of an experiment.
6. The word **'experiment'** means an operation which can produce some well defined outcome(s).

   There are two types of experiments:

   i. **Deterministic experiments:** Experiments which are repeated under identical conditions produce the same results or outcomes are called deterministic experiments.

   ii. **Random or Probabilistic experiment**: If an experiment, when repeated under identical conditions, do not produce the same outcome every time but the outcome in a trial is one of the several possible outcomes, then it is known as a random or probabilistic experiment.

   In this chapter, the term experiment will stand for random experiment.

7. The collection of all possible outcomes is called the **sample space**.
8. An outcome of a random experiment is called an **elementary event**.
9. An event associated to a random experiment is a **compound event** if it is obtained by combining two or more elementary events associated to the random experiment.
10. An event *A* associated to a random experiment is said to occur if any one of the elementary events associated to the event *A* is an outcome.
11. An elementary event is said to be **favorable** to a compound event *A*, if it satisfies the definition of the compound event *A*. In other words, an elementary event *E* is favorable to a compound event *A*, if we say that the event *A* occurs when *E* is an outcome of a trial.
12. In an experiment, if two or more events have equal chances to occur or have equal probabilities, then they are called **equally likely events**.
13. The **theoretical probability (also called classical probability) of an event *E***, written as *P* (*E*), is defined as

$$\frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

14. For two events A and B of an experiment:

    If *P(A)* > *P(B)* then event *A* is more likely to occur than event *B*.

    If *P(A)* = *P(B)* then events *A* and *B* are equally likely to occur.

15. An event is said to be **sure event** if it always occur whenever the experiment is performed. The probability of sure event is always one. In case of sure event elements are same as the sample space.

16. An event is said to be **impossible event** if it never occur whenever the experiment is performed. The probability of an impossible event is always zero. Also, the number of favorable outcome is zero for an impossible event.

17. Probability of an event lies between 0 and 1, both inclusive, i.e., $\mathbf{0 \leq P\ (A) \leq 1}$

18. If E is an event in a random experiment then the event 'not *E*' (denoted by $\bar{E}$) is called the **complementary event** corresponding to *E*.

19. The **sum of the probabilities** of all elementary events of an experiment is 1.

20. For an event *E*, $P(\bar{E}) = 1 - P(E)$, where the event $\bar{E}$ representing 'not *E*" is the complement of event E.

21. Suits of Playing Card
A pack of playing cards consist of 52 cards which are divided into 4 suits of 13 cards each. Each suit consists of one ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are named as spades, hearts, diamonds and clubs.

22. Face Cards
King, queen and jack are face cards.

23. The formula for finding the **geometric probability** of an event is given by:

$$P(E) = \frac{\text{Measure of the specified part of the region}}{\text{Measure of the whole region}}$$

Here, 'measure' may denote length, area or volume of the region or space.

MATHS

$(a+b)^2$

ab+

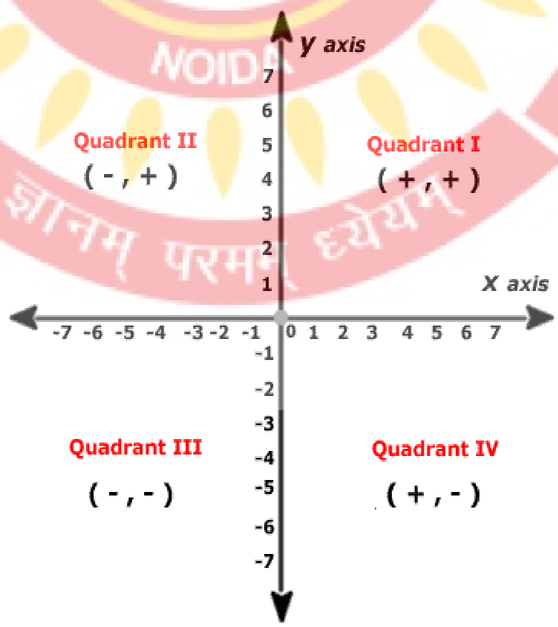
Coordinate Geometry

1. Coordinate axes:
Two perpendicular number lines intersecting at point zero are called **coordinate axes**. The point of intersection is called **origin** and denoted by '*O*'. The horizontal number line is the ***x*-axis** (denoted by *X'OX*) and the vertical one is the ***y*-axis** (denoted by *Y'OY*).

2. **Cartesian plane** is a plane formed by the coordinate axes perpendicular to each other in the plane. It is also called as *xy* plane.
The axes divide the Cartesian plane into four parts called the **quadrants** (one fourth part), numbered I, II, III and IV anticlockwise from *OX*.

3. Coordinates of a point:
- The x-coordinate of a point is its perpendicular distance from *y*-axis, called **abscissa.**
- The y-coordinate of a point is its perpendicular distance from *x*-axis, called **ordinate**
- If the abscissa of a point is *x* and the ordinate of the point is *y*, then (*x, y*) is called the **coordinates** of the point.
- The point where the *x*-axis and the *y*-axis intersect is represented by the coordinate point (0, 0) and is called the **origin**.

4. Sign of the coordinates in the quadrants:
Sign of coordinates depicts the quadrant in which it lies.
- The point having both the coordinates positive i.e. of the form (+, +) will lie in the first quadrant.
- The point having x-coordinate negative and y-coordinate positive i.e. of the form (-, +) will lie in the second quadrant.
- The point having both the coordinates negative i.e. of the form (-, -) will lie in the third quadrant.
- The point having x-coordinate positive and y-coordinate negative i.e. of the form (+,-) will lie in the fourth quadrant.



5. Coordinates of a point on the x-axis or y-axis:
The coordinates of a point lying on the *x*-axis are of the form (*x*, 0) and that of the point on the *y*-axis are of the form (0, *y*).

6. Distance formula

The distance formula is used to find the distance between two any points say $P(x_1, y_1)$ and $Q(x_2, y_2)$ which is given by: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

- The distance of a point P(x, y) from the origin O(0, 0) is $OP = \sqrt{x^2 + y^2}$.
- The points *A, B* and *C* are **collinear** if *AB* + *BC* = *AC*.

7. Determining the type of triangle using distance formula

i. Three points *A, B* and *C* are the vertices of an **equilateral triangle** if *AB* = *BC* = *CA*.
ii. The points *A, B* and *C* are the vertices of an **isosceles triangle** if *AB* = *BC or BC* = *CA or CA* = *AB*.
iii. Three points *A, B* and *C* are the vertices of *a* **right triangle** if the sum of the squares of any two sides is equal to the square of the third side.

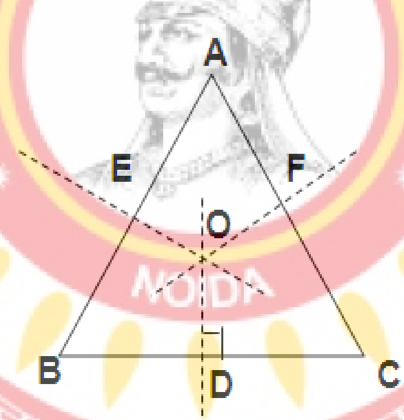
8. Determining the type of quadrilateral using distance formula

For the given four points A, B, C and D, if:

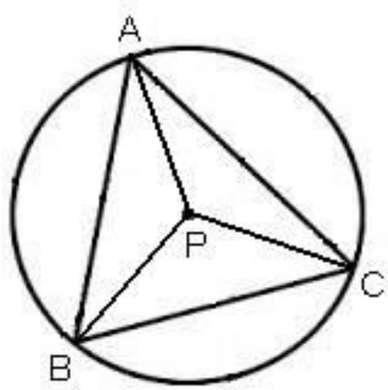
i. $AB = CD$, $BC = DA$; $AC \neq BD \Rightarrow ABCD$ is a parallelogram.
ii. $AB = BC = CD = DA$; $AC \neq BD \Rightarrow ABCD$ is a rhombus
iii. $AB = CD$, $BC = DA$; $AC = BD \Rightarrow ABCD$ is a rectangle
iv. $AB = BC = CD = DA$; $AC = BD \Rightarrow ABCD$ is a square.

9. Circumcentre of a triangle

The point of intersection of the perpendicular bisectors of the sides of a triangle is called the **circumcentre**. In the figure, O is the circumcentre of the triangle ABC.

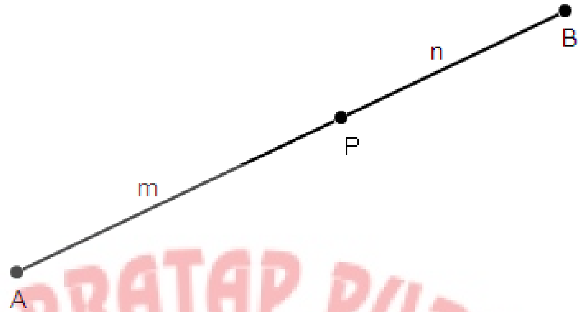


- Circumcentre of a triangle is equidistant from the vertices of the triangle. That is, ***P* is the circumcentre of Δ *ABC*, if *PA* = *PB* =*PC*.**
- Moreover, if a circle is drawn with *P* as centre and PA or PB or PC as radius, the circle will pass through all the three vertices of the triangle. *PA* (or *PB* or *PC*) is said to be the **circumradius** of the triangle.



10. Section formula

If *P* is a point lying on the line segment joining the points *A* and *B* such that *AP: BP = m: n.* Then, we say that the **point *P* divides the line segment *AB* internally** in the ratio *m: n*.

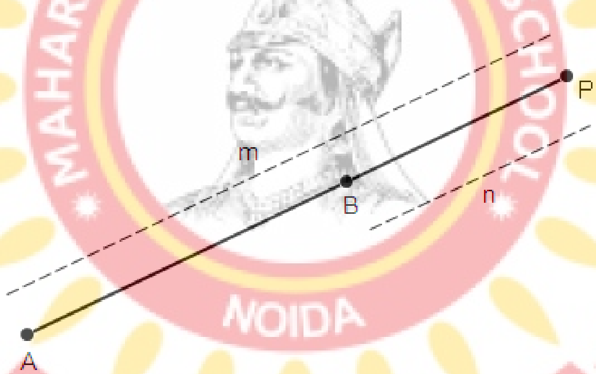


Coordinates of a point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio *m*: *n* internally are given by:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

This is known as the **section formula**.

11. If *P* is a point lying on *AB* produced such that *AP: BP = m: n*, then point ***P* divides *AB* externally** in the ratio *m: n*.



If *P* divides the line segment joining the points A (x_1, y_1) and *B* (x_2, y_2) in the ratio *m*: *n* externally, then the coordinates of point *P* are given by $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$.

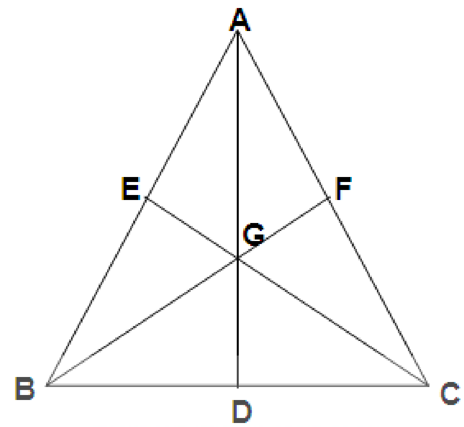
12. Coordinates of Mid-point

Mid-point divides the line segment in the ratio 1:1. Coordinates of the mid-point of a line segment joining the points (x_1, y_1) and (x_2, y_2) are

$$\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right).$$

13. Centroid of a triangle

The point of intersection of the three medians of a triangle is called the centroid.



In the figure, G is the centroid of the triangle ABC where AD, BF and CE are the medians through A, B and C respectively.
Centroid divides the median in the ratio of 2:1.

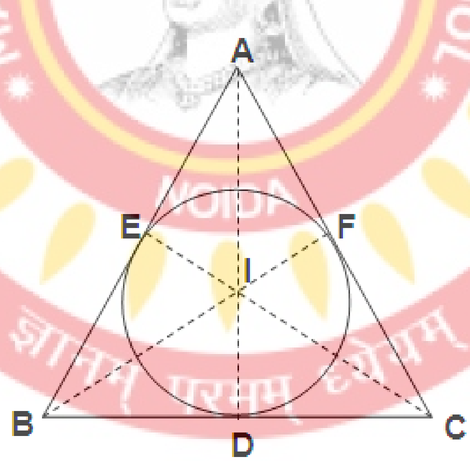
14. Coordinates of the centroid

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle ABC, then the **coordinates of the centroid** are given by

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

15. Incentre of a triangle

The point of intersection of all the internal bisectors of the angles of a triangle is called the **incentre**. It is also the centre of a circle which touches all the sides of a triangle (such type of a circle is named as the incircle).



In the figure, *I* is the incentre of the triangle ABC.

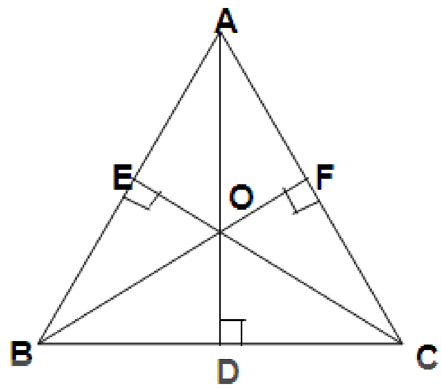
16. Coordinates of incentre

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then the **coordinates of incentre** are given by

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + ay_2 + ay_3}{a + b + c} \right)$$

17. Orthocentre of a triangle

The point of intersection of all the perpendiculars drawn from the vertices on the opposite sides (called altitudes) of a triangle is called the **Orthocentre** which can be obtained by solving the equations of any two of the altitudes.



In the figure, O is the orthocentre of the triangle ABC.

18. If the triangle is equilateral, the centroid, the incentre, the orthocenter and the circumcentre coincides.

19. Orthocentre, centroid and circumcentre are always collinear, whereas the centroid divides the line joining the orthocentre and the circumcentre in the ratio of 2:1.

20. Area of a triangle

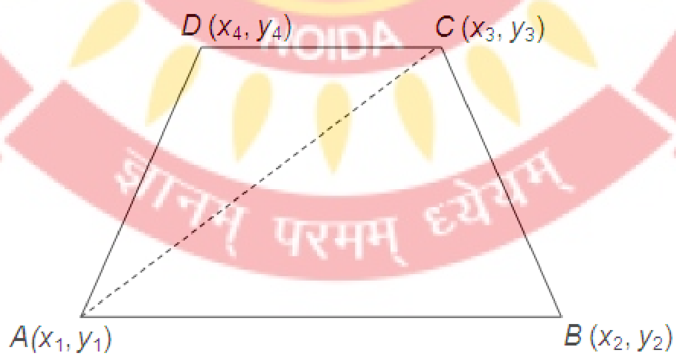
If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then the **area of triangle** ABC is given by $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$.

- Three given points are **collinear**, if the **area of triangle formed by these points is zero.**

21. Area of a quadrilateral

Area of a quadrilateral can be calculated by dividing it into two triangles.

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD



Note: To find the area of a polygon, divide it into triangular regions having no common area, then add the areas of these regions.

MATHS

Constructions

1. **To divide a line segment internally in a given ratio m: n**, where both m and n are positive integers, we follow the steps given below:

Step 1: Draw a line segment *AB* of given length by using a ruler.

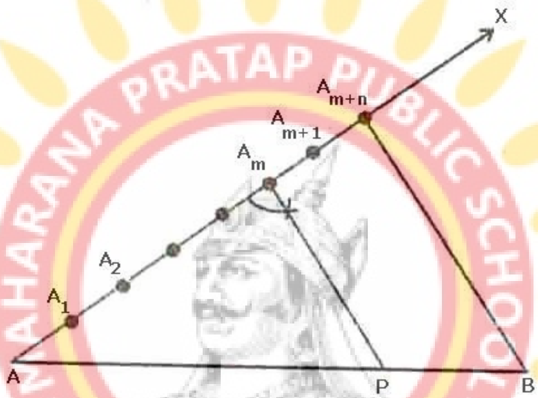
Step 2: Draw any ray *AX* making an acute angle with *AB*.

Step 3: Along *AX* mark off $(m + n)$ points $A_1, A_2, \ldots\ldots A_{m-1}, A_{m+1}, \ldots\ldots, A_{m+n}$, such that $AA_1 = A_1A_2 = A_{m+n-1}A_{m+n}$.

Step 4: Join BA_{m+n}

Step 5: Through the point A_m, draw a line parallel to $A_{m+n}B$ by making an angle equal to $\angle AA_{m+n}B$ at A_m, intersecting AB at point P.

The point *P* so obtained is the required point which divides *AB* internally in the ratio *m: n*.



Justification

In ΔABA_{m+n}, we observe that A_mP is parallel to $A_{m+n}B$. Therefore, by Basic Proportionality theorem, we have:

$$\frac{AA_m}{A_mA_{m+n}} = \frac{AP}{PB}$$

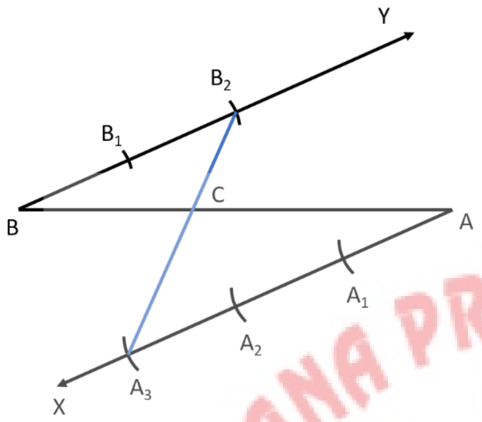
$$\Rightarrow \frac{AP}{PB} = \frac{m}{n} \qquad \left[\because \frac{AA_m}{A_mA_{m+n}} = \frac{m}{n}, \text{ by construction}\right]$$

$$\Rightarrow AP : PB = m : n$$

Hence, *P* divides *AB* in the ratio *m: n.*

2. Alternative method to divide a line segment internally in a given ratio m: n

Example
Find the point C such that it divides BA in ratio 2:3



Steps of Construction :
1. Draw any ray XA making an acute angle with BA.
2. Draw a ray YB parallel to XA by making ∠YBA equal to ∠XAB.
3. Locate the points A_1, A_2, A_3 (m = 3) on AX and B_1, B_2 (n = 2) on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$.
4. Join A_3B_2. Let it intersect AB at a point C
   Then BC : CA = 2:3

Justification

Here $\Delta BB_2C \sim \Delta AA_3C$ …AA test

$$\frac{BB_2}{AA_3} = \frac{BC}{AC} \ldots \text{(c.p.s. t .)}$$

$$\frac{2}{3} = \frac{BC}{AC}$$

3. The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as a **scale factor**. The scale factor may be less or greater than 1.

4. If the scale factor is less than 1, then the new figure will be smaller in comparison to the given figure.

5. If the scale factor is greater than 1, then the new figure will be bigger in comparison to the given figure.

Construction of Triangle Similar to given Triangle

Consider a triangle *ABC*. Let us construct a triangle similar to ΔABC such that each of its sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding sides of ΔABC.

Steps of constructions when $m < n$:

Step 1: Construct the given triangle *ABC* by using the given data.

Step 2: Take any one of the three side of the given triangle as base. Let *AB* be the base of the given triangle.

Step 3: At one end, say *A*, of base *AB*. Construct an acute angle $\angle BAX$ below the base *AB*.

Step 4: Along *AX* mark off *n* points $A_1, A_2, A_3, \ldots\ldots, A_n$ such that

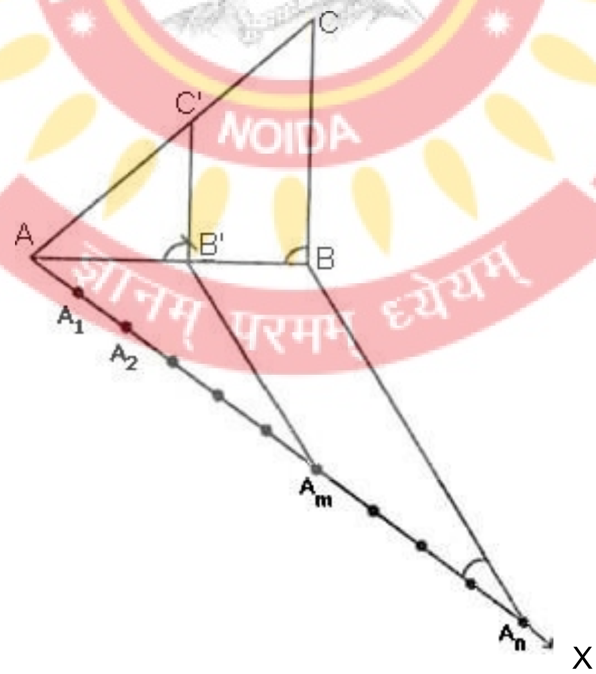
$$AA_1 = A_1A_2 = \ldots\ldots = A_{n-1}A_n$$

Step 5: Join A_nB

Step 6: Draw A_mB' parallel to A_nB which meets *AB* at *B*'.

Step 7: From *B*' draw $B'C' \| BC$ meeting *AC* at *C*'.

Triangle *AB*'*C*' is the required triangle each of whose sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding side of ΔABC.



Justification

Since $A_m B' \| A_n B$. Therefore

$$\frac{AB'}{B'B} = \frac{AA_m}{A_m A_n}$$

[by basic proportionality theorem]

$$\Rightarrow \frac{AB'}{B'B} = \frac{m}{n-m}$$

$$\Rightarrow \frac{B'B}{AB'} = \frac{n-m}{m}$$

Now $\frac{AB}{AB'} = \frac{AB' + B'B}{AB'}$

$$\Rightarrow \frac{AB}{AB'} = 1 + \frac{B'B}{AB'} = 1 + \frac{n-m}{m} = \frac{n}{m}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{m}{n}$$

In triangles ABC and $AB'C'$, we have

$$\angle BAC = \angle B'AC'$$

and $\angle ABC = \angle AB'C'$

So, by AA similarity criterion, we have

$$\Delta AB'C' \sim \Delta ABC$$

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{m}{n}$$

Steps of construction when m > n:

Step 1: Construct the given triangle by using the given data.

Step 2: Take any one of the three sides of the given triangle and consider it as the base. Let *AB* be the base of the given triangle.

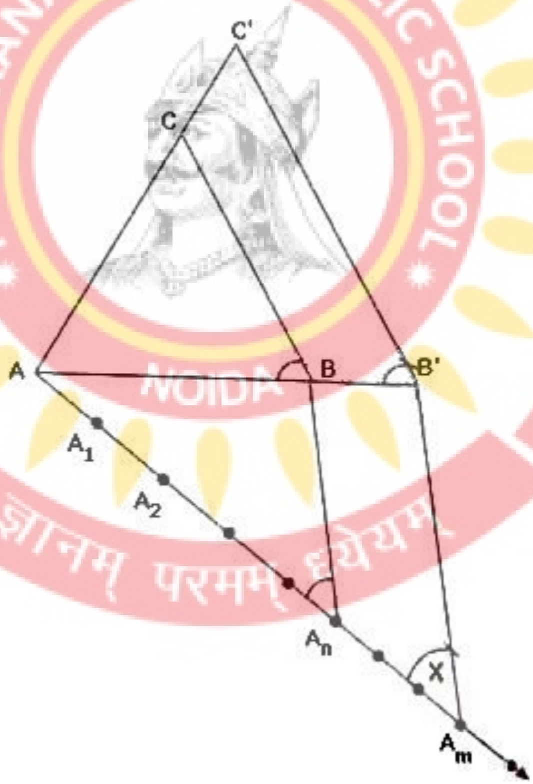
Step 3: At one end, say *A,* of base *AB*. Construct an acute angle $\angle BAX$ below base AB i.e., on the opposite side of the vertex *C*.

Step 4: Along *AX* mark off *m* (large of m and n) points $A_1, A_2, A_3, \ldots\ldots\ldots A_m$ of *AX* such that $AA_1 = A_1A_2 = \ldots\ldots\ldots = A_{m-1}A_m$.

Step 5: Join A_nB to *B* and draw a line through A_m parallel to A_nB, intersecting the extended line segment *AB* at *B*'.

Step 6: Draw a line through *B*' parallel to *BC* intersecting the extended line segment *AC* at *C*'.

Step 7: $\Delta AB'C'$ so obtained is the required triangle.



Justification

Consider triangle *ABC* and *AB' C'*. We have:

$$\angle BAC = \angle B'AC'$$

$$\angle ABC = \angle AB'C'$$

So, by *AA* similarity criterion,

$$\Delta ABC \sim \Delta AB'C'$$

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$$

$In\ \Delta A\ A_m B',\ A_n B \parallel A_m B'.$

$$\therefore \frac{AB}{BB'} = \frac{AA_n}{A_n A_m}$$

$$\Rightarrow \frac{BB'}{AB} = \frac{A_n A_m}{AA_n}$$

$$\Rightarrow \frac{BB'}{AB} = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB' - AB}{AB} = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB'}{AB} - 1 = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{m}{n}$$

From (i) and (ii), we have

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{m}{n}$$

The tangent to a circle is a line that intersects the circle at exactly one point.

Tangent to a circle is perpendicular to the radius through the point of contact.

Construction of Triangle to a Circle from a point outside the Circle

Construction of a tangent to a circle from a point outside the circle, when its centre is known

The steps of constructions are as follows:

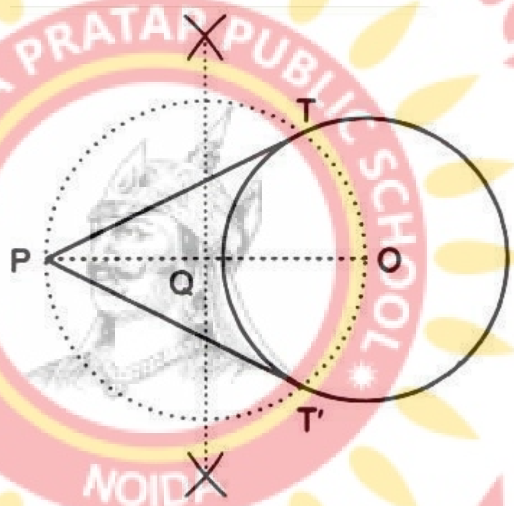
Step 1: Join the centre *O* of the circle to the point *P*.

Step 2: Draw perpendicular bisector of *OP* intersecting *OP* at *Q*.

Step 3: With *Q* as centre and radius *OQ*, draw a circle. This circle has *OP* as its diameter.

Step 4: Let this circle intersect the first circle at two points *T* and *T'*. Join *PT* and *P T'* .

PT and *P T'* are the two tangents to the given circle from the point P.



Justification

Join *OT* and *O T'*

It can be seen that $\angle PTO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle PTO = 90°$

$\Rightarrow OT \perp PT$

Since *OT* is the radius of the circle, *PT* has to be a tangent of the circle. Similarly, *PT'* is a tangent of the circle.